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MULTIATTRIBUTE METHODOLOGIES FOR
DECISION MAKING IN COEAS

FINAL REPORT

May 1992

DEPARTMENT OF THE ARMY
HEADQUARTERS UNITED STATES ARMY TRAINING AND DOCTRINE COMMAND
FORT MONROE, VIRGINIA 23651

TRADOC ANALYSIS COMMAND
Fort Benjamin Harrison & White Sands Missile Range

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SECURITY CHECKLIST

1. TITLE OF STUDY: Multiattribute Methodologies for Decision Making in COEAs.
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ABSTRACT

This report encompasses discussion and illustration of operations research techniques for integrating multiple decision attributes within Cost and Operational Effectiveness Analyses (COEA). It posits criteria for selecting techniques that may be useful in conducting COEA, concludes that five techniques are particularly worthy of COEA practitioner consideration, and then illustrates the use of two techniques--the Analytic Hierarchy Process (AHP) and Multiattribute Utility Theory (MAUT).

**Multiattribute Methodologies for
Decision Making in COEAs**

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Multiattribute Methodologies for Decision Making in COEAs

1. Purpose. This report documents the Multiattribute Methodologies for Decision Making in COEAs study. The study was conducted in partnership between TRAC-FBHN and TRAC-WSMR and sponsored by the office of the Director of MANPRINT, HQ DA.

2. References. See Appendix A.

3. Terms of Reference.

a. Background. This study discusses and illustrates the use of well accepted operations research techniques for integrating multiple decision attributes, often conflicting, within the context of Cost and Operational Effectiveness Analyses(COEA). As an oversimplified example, consider a case in which the Army is going to buy one of three alternative helicopters. The decision as to which one to buy may depend on any number of attributes. Attributes are the criteria decision makers consider in making choices. In COEA they might include cost, combat effectiveness, supportability, manpower, safety, and the like. Typically COEA focus on the assessment of the alternatives with regard to specified attributes. The emphasis is on estimating cost, combat effectiveness, supportability, manpower, safety, and so on. There is often little or no attempt to integrate the attribute estimates or outcomes into an overall scheme to further guide the decision maker in the choice of alternatives.

b. Problem. There is presently a lack of well accepted techniques within the Army analytic community for integration of distinct and often conflicting attributes within COEA and other studies. When analysis delineates alternatives and leads to recommendations to Army decision makers, such analysis should be consistent with state of the art operations research techniques so as to be understandable, verifiable, repeatable, and robust to criticism from members of the analytic community. Present Army analysis with respect to integration of multiple attributes frequently fails to meet these standards.

c. Impact of Problem. The disuse of well accepted, practical integration techniques makes trade-offs among multiple attributes awkward and sometimes controversial. Trade-offs may often defy a ready understanding by the target audience or the decision maker.

d. Objectives. This study has the following two objectives:

(1) To delineate and summarize practical multiattribute integration techniques for use in COEA.

(2) To demonstrate the use of two of the most promising multiattribute techniques in COEA.

e. Essential Elements of Analysis (EEA). This study has the following EEA:

EEA 1. What are the principal multiattribute techniques for integrating attributes in the choice of alternatives?

EEA 2. What are the most practical multiattribute techniques for use in COEA?

EEA 3. How do the most practical techniques compare in a typical COEA application?

f. Scope. This study considers the following decision situation. There are a small number of alternatives, say no more than 30, which are to be compared with regard to a small number of attributes, also say no more than 30. Outcomes for the alternatives with regard to the attributes have been estimated. Techniques under consideration are designed to integrate the various estimated outcomes into an overall ranking of the alternatives.

(1) Integration techniques under consideration are referred to in the literature with the terms multiattribute (multiple attribute), multiobjective (multiple objective), multidimensional (multiple dimensional), and multicriteria (multiple criteria). The term used herein is multiattribute.

(2) Multiattribute integration techniques are distinguished from optimization techniques, such as linear programming, integer programming, or goal programming. Multiattribute techniques focus on the determination of the integration process, the optimization process being trivial. In programming the determination of the integration process, the objective function, is secondary to the nontrivial optimization over an infinite or at an least extremely large number of alternatives.

g. Limitations.

(1) Multiattribute techniques are often roughly categorized as either descriptive or prescriptive. Descriptive techniques derive from research into behavioral decision theory which seeks primarily to describe how people make decisions. Prescriptive techniques, the concern of operations research, seek to help people make better decisions. Prescriptive techniques are founded on compelling axioms or practical decision criteria whereas descriptive techniques rely on empirical decision data. This study focuses on prescriptive techniques. Consequently, techniques such as a regression model based on historical decision data, Elimination by Aspects (EBA), Linear Programming Techniques for Multidimensional Analysis of Preference (LINMAP), and Maximum Likelihood Hierarchical (MLH) are not considered.

(2) While the operations research literature abounds with theoretically sound or practical techniques for the individual decision maker, the same cannot be said for the decision making group. Techniques considered in this study were designed primarily for use by a single decision maker. However, some, such as the

Analytic Hierarchy Process (AHP), may be highly adaptable to a decision making group.

(3) Of the techniques identified in the compendium, several are selected as appropriate for general use in COEA. Factors used to determine appropriateness include theoretical soundness, applicability to the COEA process, ease of use, availability of application software, data requirements, and prevalence (which encompasses demonstrated effectiveness). The criteria serve as a general screen for techniques. They are not independent, nor are they necessarily criteria that others may invoke. They do strike us as reasonable for present circumstances. The purpose of this paper is not to evaluate explicitly the techniques against the criteria, but rather to set out viable techniques as well as some illustration of their use. Nonetheless, for the sake of clarity, we offer the following explanation of our criteria:

By theoretical soundness, we mean the extent to which a technique has successfully withstood scrutiny in refereed journals.

By applicability to the COEA process, we mean the apparent match between what a technique purports to accomplish and the purpose of a COEA (i.e., "to select the preferred alternative").

By ease of use, we mean the degree of difficulty involved in gathering required data, employing software, explicating outcomes, and the like.

By availability of application software, we mean ready accessibility of electronic aids for performing a technique (which might be tedious if performed by hand).

By data requirements, we mean the extent to which a technique calls for data that may be difficult to obtain or process.

By prevalence, we mean an evolving history of use in study applications and/or appearance of the technique is refereed journals.

Of the acceptable techniques several are illustrated on COEA as permitted by the study resource constraints.

h. Assumptions. Multiattribute techniques enable trade-offs which are consistent with decision makers' preferences. To accomplish this, the techniques often rely on data directly from the decision makers. One must assume that decision makers will be available to provide data as necessary.

i. Constraints. Techniques are demonstrated on a recently completed COEA. Because the demonstrations are ancillary to and separate from the primary COEA effort, data required by the techniques may not be readily available. Given such a situation and a desire to use the techniques, parametric analysis or output to input analysis may be conducted. Finally, no criticism of analytical procedure in the selected COEA is made or implied. Resources are constrained to one professional staff year to be expended in no more than one calendar year.

j. Alternative Techniques.

(1) Table 1 list the multiattribute techniques considered in this study.

Table 1
Multiattribute Techniques

Dominance
Maximin
Maximax
Majority Rule
Koler's Ranking Technique
Conjunctive Technique
Disjunctive Technique
Stochastic Dominance
Lexicographic
Lexicographic with Minima
Key Attribute
ELECTRE
Permutation
Analytic Hierarchy Process (AHP)
Multiattribute Value Theory (MAVT)
Simple Additive Weighting (SAW)
Technique for Order Preference by
Similarity to Ideal Solution (TOPSIS)
Cost-effectiveness ratios
Multiattribute Utility Theory (MAUT)

These techniques are summarized in Appendix B. In accordance with the limitations of the study the techniques listed in Table 2 are found appropriate for general use in COEA. Note that the term "theory" herein connotes a technique.

Table 2
Techniques Appropriate for General
Use in COEA

Dominance (including stochastic)
Conjunctive technique
Analytic Hierarchy Process (AHP)
Multiattribute Value theory (MAVT)
Multiattribute Utility theory (MAUT)

The fact that a technique is not found appropriate for general use does not mean that the technique has no worth. In particular simple ranking techniques such as majority rule and Koler's ranking technique have great potential for use in COEAs. However, further research is required for a full understanding for their application in a COEA environment. On the other hand, techniques such as cost-effectiveness ratios or TOPSIS, are not considered appropriate for general use in COEAs because of their unique underlying

assumptions. If analysis shows that these unique assumptions hold for a particular application, then their use should be considered.

(2) Because of its rationality and ease of use, dominance should always be applied in a COEA. However, it is most likely that dominated alternatives have fallen out of consideration before the COEA. Practically, this technique would serve only as a screening device to determine acceptable (non-dominated) and unacceptable (dominated) alternatives. If true minimum acceptable levels have been set, e.g., as in an Operational Requirements Document (ORD), the conjunctive technique should also be applied to screen out any unacceptable alternatives.

(3) From the above list, we demonstrate AHP and MAUT. Since analysts typically view utility theory as the stochastic analogue of value theory, and since resource constraints preclude the demonstration of both value theory and utility theory, utility theory (MAUT) was selected since it is the more general of the two.

4. Structuring the Decision Problem.

a. Defining the Problem.

(1) The multiattribute techniques are demonstrated in conjunction with recent TOW Sight Improvement Program (TSIP) Abbreviated Cost and Operational Effectiveness Analysis (COEA). This study was conducted to scrutinize alternative antiarmor capabilities in mechanized infantry and light infantry units. The demonstration will consider only a subset of alternatives from the mechanized infantry portion of the COEA. The mainstay of the current antiarmor capability of these units is the tube-launched, optically-tracked, wire-guided (TOW) missile. This missile will be upgraded to the TOW 2B configuration. In addition, the TOW sight improvement program seeks to replace the current sight with a new sight.

(2) Mechanized infantry battalions include four mechanized infantry companies fielding Bradley Fighting Vehicles (BFV) firing TOW missiles. In addition, the battalion contains an antiarmor company with improved TOW vehicles (ITV), also firing TOW.

b. Alternatives. The infantry antiarmor COEA considered improving either the sight of the ITV TOW or the sight of the BFV TOW or sights of both. Table 3 lists the four demonstration alternatives.

Table 3
Alternatives

	Mech Inf Co BFV TOW Sight	Antiarmor Co ITV TOW Sight Alternative
Base Case	Current	Current
Alt 1	Current	New
Alt 2	New	Current
Alt 3	New	New

c. Attributes. The development of study attributes begins with the specification of the following very broad decision criterion: "to efficiently provide an infantry antiarmor capability." Working from this broad criterion, a number of more specific decision criteria were delineated. A preliminary list of these criteria appear in Table C-1 of Appendix C. This process of refinement continued until a set of quantifiable attributes was developed. These attributes, which are intended to capture as completely as possible all decision criteria, are broadly categorized as benefit attributes or resource attributes. These attributes are listed in Table 4. There is a single resource or cost attribute which is a summation of five components. These parallel the five components typically found in COEA cost analysis. The scale for this attribute is FY92 constant dollars with a twenty year time horizon. For purposes of this study, the sustainment component includes manpower, personnel, and training resource impacts.

Table 4
Attributes

Benefit attributes:

1. Ratio of red losses to blue losses for European brigade meeting engagement
2. Ratio of red losses to blue losses for European balance task force defense
3. Ratio of red losses to blue losses for Southwest Asian brigade meeting engagement

Resource attribute (summation of five components)

- a. Development Costs
- b. Production Costs
- c. Military Construction Costs
- d. Fielding Costs
- e. Sustainment Costs (including Manpower)

(1) Some decision criteria could be treated in either a benefit (output) or resource (cost, input) fashion. For example, consider air transportation as a factor. COEA alternatives may be such that some are air transportable and some are not. Consequently the binary attribute, "air transportable" (yes or no), could be treated as a benefit attribute. On the other hand, air transportability might be a requirement specified in the system ORD. In this case, air transportability could be captured in increased development, production, or supportability costs of the alternatives. For the purposes of the demonstration, all sustainment or supportability and MANPRINT criteria are handled on the resource or cost side. For example, system safety is assumed specified at a certain level, with differences among alternatives possibly showing as differences in development, production, or sustainment.

(2) In developing attributes, the priority of the criteria and the comprehensiveness of the assessment modeling must be considered. For example, consider the criteria "combat effectiveness" and "equipment availability." Availability alone is not important. It is important only through its influence on combat effectiveness. Combat effectiveness may be assessed with a combat model that is capable of representing the effect of alternatives with differing availability rates. Thus, availability may be subsumed under the combat effectiveness attributes.

(3) The benefits of alternatives are assessed using attrition measures from a high resolution combat simulation model. Three different combat scenarios are represented. For each scenario the loss exchange ratio (LER), red losses divided by blue losses, is an attribute. The three scenarios, "European brigade meeting engagement," "European balanced task force defense," and "Southwest Asian (SWA) brigade meeting engagement," pit 1996 projected blue forces against 2004 projected red forces. Descriptions of the scenarios appear in Appendix C. The fact that the attributes are based on only three scenarios does not mean that conflict is contemplated in only these three narrowly defined situations. Rather, these scenarios are viewed as representative of conflicts that might actually occur.

5. Demonstration of Analytic Hierarchy Process

a. For the demonstration of AHP Figure 1 illustrates the COEA attributes in a hierarchical fashion.

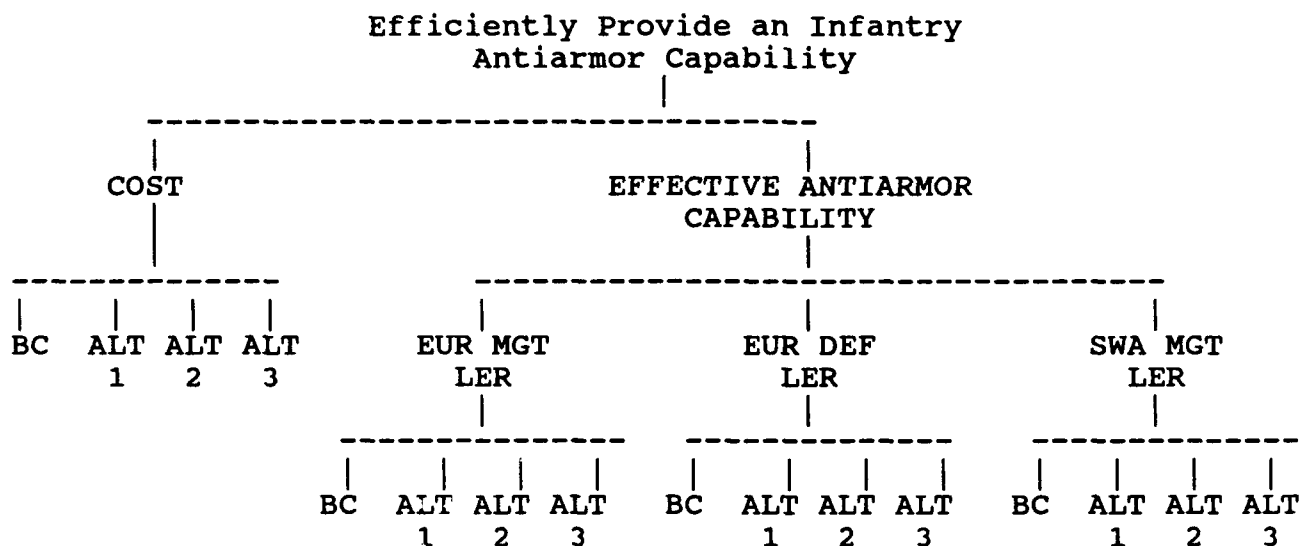


Figure 1. The Attribute Hierarchy

The hierarchy requires six distinct scalings. At the top the decision maker provides scale values on the relative importance of cost and effectiveness in meeting the overall decision criterion. Continuing down the hierarchy, the decision maker scales the relative importance of the three LER attributes in terms of their contribution to the effectiveness attribute. Next the alternatives are scaled in terms of their contribution to the cost attribute, the European meeting LER attribute, the European Defense LER attribute, and the SWA meeting LER attribute.

b. For demonstration purposes hypothetical scale values are used. While we feel that these are realistic values, we do not claim empirical validity. The AHP scale values are defined in Table 2 of Appendix B. The eigenvector weights of the respective scale value matrices are computed using a FORTRAN subroutine from the IMSL library for mathematical applications.

(1) Table 5 gives the scale values for the relative importance of the contribution of cost and antiarmor capability to the decision. This choice of values weights cost and effectiveness equally.

Table 5
Pairwise Comparisons of
Cost and Antiarmor Capability

	Cost	Antiarmor Capability
Cost	1	1
Antiarmor Capability	1	1

Eigenvector weightings - (0.50, 0.50)

(2) Table 6 gives the scale values for the relative importance of the three LER attributes in their contribution to the antiarmor capability. The scale values have been chosen so that the first two LER attributes are equally valued and the third is weakly more important.

Table 6
Pairwise Comparisons of
LER Attributes

	EUR MGT LER	EUR DEF LER	SWA MGT LER
EUR MGT LER	1	1	1/3
EUR DEF LER	1	1	1/3
SWA MGT LER	3	3	1

Principal Eigenvalue - 3.00

Eigenvector weighting - (0.20, 0.20, 0.60)

A principal eigenvalue of 3.00 indicates that the scale values are completely consistent.

(3) Table 7 gives the scale values for the cost of the alternatives. These scale values are derived from cost estimates (in FY92 constant million dollars) presented in Table C-2 of Appendix C. The scale values have been chosen so that the base case is valued strongly over alternatives one and two, alternatives one and two are equally valued, and alternatives one and two are valued strongly over alternative three. In addition the base case is valued absolutely over alternative three. The "cheaper is better" principle applies.

Table 7
Pairwise Comparisons of
Cost of Alternatives

	Base Case	Alt 1	Alt 2	Alt 3
Cost	350	900	982	1344
Base Case	1	5	5	9
Alt 1	1/5	1	1	5
Alt 2	1/5	1	1	5
Alt 3	1/9	1/5	1/5	1

Principal Eigenvalue - 4.13

Eigenvector weighting - (0.64, 0.16, 0.16, 0.04)

A principal eigenvalue of 4.13 indicates that the scale values are highly consistent.

(4) Tables 8, 9, and 10 give scale values of the alternatives for the LER attributes. Also these tables give the eigenvector weightings and principal eigenvalues. These scale values are based on the median LER estimates from Tables C-6, C-7, and C-8 of Appendix C. In Table 8 the base case and alternative one are valued the same. Alternative two is valued strongly over the base case and alternative one. Alternative three is valued strongly over alternative two. Also alternative three is valued absolutely over the base case and alternative one. In contrast to cost, higher LERs are "better."

Table 8
Pairwise Comparisons of
European Meeting LER Estimates

	Base Case	Alt 1	Alt 2	Alt 3
Median LER	1.00	1.04	1.40	1.77
Base Case	1	1	1/5	1/9
Alt 1	1	1	1/5	1/9
Alt 2	5	5	1	1/5
Alt 3	9	9	5	1

Principal Eigenvalue - 4.13

Eigenvector weighting - (0.06, 0.06, 0.22, 0.66)

A principal eigenvalue of 4.13 indicates that the scale values are highly consistent.

In Table 9 alternatives one and two are equally valued. Alternatives one and two are valued weakly over the base case and alternative three is value weakly over alternatives one and two. Also alternative three is valued strongly over the base case.

Table 9
Pairwise Comparisons of
European Defense LER Estimates

	Base Case	Alt 1	Alt 2	Alt 3
Median LER	2.29	2.44	2.38	2.57
Base Case	1	1/3	1/3	1/5
Alt 1	3	1	1	1/3
Alt 2	3	1	1	1/3
Alt 3	5	3	3	1

Principal Eigenvalue - 4.04

Eigenvector weighting - (0.08, 0.20, 0.20, 0.52)

A principal eigenvalue of 4.04 indicates that the scale values are highly consistent.

In Table 10 the base case and alternative one are equally valued. Alternative two is valued weakly over the base case and alternative one. Alternative three is valued weakly over alternative two and strongly over the base case and alternative one.

Table 10
Pairwise Comparisons of
SWA Meeting LER Estimates

	Base Case	Alt 1	Alt 2	Alt 3
Median LER	1.34	1.37	1.49	1.62
Base Case	1	1	1/3	1/5
Alt 1	1	1	1/3	1/5
Alt 2	3	3	1	1/3
Alt 3	5	5	3	1

Principal Eigenvalue - 4.04

Eigenvector weighting - (0.10, 0.10, 0.24, 0.56)

A principal eigenvalue of 4.04 indicates that the scale values are highly consistent.

d. The final step in the AHP is to aggregate the separate weightings into an overall weighting for each alternative. The

following linear format, described in Appendix B, is used.

$$\begin{aligned} \text{AHP Weighting} = & 0.5 * (\text{Cost Weighting}) \\ & + 0.5 * 0.2 * (\text{EUR MGT LER Weighting}) \\ & + 0.5 * 0.2 * (\text{EUR DEF LER Weighting}) \\ & + 0.5 * 0.6 * (\text{SWA MGT LER Weighting}) \end{aligned}$$

Table 11 gives the overall AHP weighting for each alternative.

Table 11
AHP Weightings of Alternatives

Alternative	Weighting
Base Case	0.36
Alt 1	0.13
Alt 2	0.20
Alt 3	0.31

The base case ranks first followed by alternative three. The greater effectiveness of alternative three is offset by its greater cost. This results in part from valuing the importance of cost and effectiveness equally. Had effectiveness been valued just very weakly over cost the scale values of Table 5 would be replaced by those of Table 12 below.

Table 12
Revised Pairwise Comparisons of
Cost and Antiarmor Capability

	Cost	Antiarmor Capability
Cost	1	1/2
Antiarmor Capability	2	1
Eigenvector weightings - (1/3, 2/3)		

With these new weightings for cost and effectiveness the overall weightings of the alternatives become those of Table 13.

Table 13
Revised AHP Weightings of Alternatives

Alternative	Weighting
Base Case	0.27
Alt 1	0.12
Alt 2	0.21
Alt 3	0.40

In this case alternative three ranks first followed by the base case. One can show with a simple computation that if effectiveness is valued at 1.21 over cost, the base case and alternative three tie for the first rank with equal weightings.

6. Demonstration of Multiattribute Utility Theory

a. As with AHP the application of MAUT begins with hierarchical decomposition illustrated in Figure 1. The complete demonstration requires three steps: modeling utility, modeling the probability distribution of outcomes, and integrating the two models.

b. Modeling utility. To simplify the demonstration, the attributes are assumed to satisfy appropriate utility independence conditions so that the multiattribute utility function has either the additive or the multiplicative form of the multilinear decomposition (refer to Appendix B Compendium). Formal checking of this assumption would be required in an actual application. Its failure to hold would require more complex techniques. Following this assumption the first step is to determine the single attribute utility functions.

(1) For demonstration purposes hypothetical functions are used. These functions are such that the decision maker is risk neutral with regard each single attribute. While one hopes that these are reasonable functions, no claim is made to their empirical validity. Table 14 gives these functions which are scaled from 0 to 1 over their range. Risk neutrality requires that these functions be linear.

Table 14
Single Attribute Utilities

Attribute	Range	Utility Constant	Scaling
Cost	200 to 1500	$UC(c) = (1500-c)/1300$	KC
EUR MGT LER	0.25 to 4.0	$UM(x) = 4(x - 0.25)/15$	KM
EUR DEF LER	0.25 to 4.0	$UD(y) = 4(y - 0.25)/15$	KD
SWA MGT LER	0.25 to 4.0	$US(z) = 4(z - 0.25)/15$	KS

Using the four scaling constants introduced in Table 14 the utility function, $U(c,x,y,z)$, decomposes as

$$\begin{aligned}
 U(c,x,y,z) = & KC*UC(c) + KM*UM(x) + KD*UD(y) + KS*US(z) \\
 & + K*(KC*KM*UC(c)*UM(x) + KC*KD*UC(c)*UD(y) + \\
 & \quad KC*KS*UC(c)*US(z) + KM*KD*UM(x)*UD(y) + \\
 & \quad KM*KS*UM(x)*US(z) + KD*KS*UD(y)*US(z)) \\
 & + K*K*(KC*KM*KD*UC(c)*UM(x)*UD(y) + \\
 & \quad KC*KM*KS*UC(c)*UM(x)*US(z) + \\
 & \quad KC*KD*KS*UC(c)*UD(y)*US(z) + \\
 & \quad KM*KD*KS*UM(x)*UD(y)*US(z)) \\
 & + K*K*K*(KC*KM*KD*KS*UC(c)*UM(x)*UD(y)*US(z))
 \end{aligned}$$

Over the range of attribute values the function U is scaled from 0 to 1. The value 0 is obtained when all attributes are at their worst value, i.e., $U(1500,0.25,0.25,0.25) = 0$. The value 1 is obtained when all attributes are at their best value, i.e., $U(200,4.0,4.0,4.0) = 1$. If the constant K in the decomposition of U is 0 the additive form holds otherwise the multiplicative form holds.

(2) In the next step values of the four scaling constants, KC through KS , are determined. This requires detailed and complex assessment sessions with the decision maker. Software such as IDEA is useful in simplifying this assessment. After the four scaling constants, KC through KS , are determined, the constant K is determined numerically as the solution of a polynomial. Table 15 gives hypothetical constant values for the demonstration. The scaling constants, KC through KS , have a utility interpretation at the extreme values of the attributes. They are not to be interpreted as the relative importance of an attribute, i.e., KC is not interpreted as the relative importance of the cost attribute.

Table 15
MAUT Scaling Constants

Constant	Utility Equivalent	Value
KC	U(200,0.25,0.25,0.25)	0.200
KM	U(1500,4.0,0.25,0.25)	0.300
KD	U(1500,0.25,4.0,0.25)	0.300
KS	U(1500,0.25,0.25,4.0)	0.400
K	not applicable	-0.409

c. Modeling Outcomes. To simplify the demonstration the attributes for any particular alternative are assumed to be probabilistically independent (refer to Appendix B Compendium). Formal checking of this assumption would be required in an actual application. Its failure to hold would require more complex techniques. Following this assumption probability distributions of outcomes for each alternative and each attribute must be developed.

(1) Because of the assumptions of independence and risk neutrality only the means of the distributions of outcomes are required. Table 16 summarizes the necessary values from Tables C-5, C-6, C-7, and C-8 of Appendix C.

Table 16
Means of Outcomes

Alternative	Attributes			
	Cost	EUR MGT LER	EUR DEF LER	SWA MGT LER
Base Case	350	1.06	2.35	1.30
Alt 1	900	1.08	2.46	1.40
Alt 2	982	1.45	2.36	1.52
Alt 3	1344	1.81	2.78	1.63

d. Integration. The last step is to combine the utility model and the probability model. This step proceeds by computing the expected utility for each alternative by integrating the multivariate utility function with the probability distribution of outcomes. The alternatives are then ranked by their expected utility. Because of the assumptions of independence and risk neutrality, the expected utility of an alternative may be computed

by substituting the appropriate mean values from Table 16 into the utility function $U(c,x,y,z)$. Table 17 gives the expected utilities for each alternative. Expected utility represents a reasonable basis for selection of an alternative by a decision maker.

Table 17
Expected Utilities

Alternative	Expected Utility
Base Case	0.483
Alt 1	0.428
Alt 2	0.447
Alt 3	0.465

As was the situation with AHP the base case ranks first followed by alternative three. If the value of KC, which equals $U(200,0.25,0.25,0.25)$, were decreased from 0.20 to 0.15, the expected utilities of Table 17 would be replaced by those of Table 18.

Table 18
Revised Expected Utilities

Alternative	Expected Utility
Base Case	0.451
Alt 1	0.413
Alt 2	0.435
Alt 3	0.466

Now alternative three ranks first followed by the base case. One can show with a simple computation that with KC valued at 0.172, the base case and alternative three tie for the first rank with equal expected utilities.

7. Summary. With reference to an attached compendium, this paper delineates and summarizes practical multiattribute integration techniques for use in COEA. Further, it provides a demonstration of two practical multiattribute techniques for use in COEA. Responses to Essential Elements of Analysis (EEAs) as well as conclusions follow.

a. Essential Elements of Analysis (EEA)

EEA 1. What are the principal multiattribute techniques for integrating attributes in the choice of alternatives?

A number of techniques exist which may be used to integrate attributes in the choice of alternatives. "Principal techniques," for the purposes of the Army decision making environment, comprise those which are (1) prescriptive in nature, and (2) are designed for a single decision maker or are easily adapted to group use. Nineteen such techniques are set out in the compendium of this report.

EEA 2. What are the most practical multiattribute techniques for use in COEA?

Five techniques stand out as probable candidates for general use in COEA. These include (1) dominance, (2) the conjunctive technique, (3) the Analytic Hierarchy Process (AHP), (4) Multiattribute Value Theory (MAVT), and (5) Multiattribute Utility Theory (MAUT). Each of these techniques rates highly on at least four of the six criteria set out earlier in this report. Dominance and the conjunctive technique, however, are perhaps not strictly accountable to the criteria of "availability of application software" or "prevalence" since application software is generally unnecessary, and since such techniques may be applied without explicit acknowledgement in final reports or journal articles. The other three techniques, in some instances, rate poorly with respect to "ease of use" and/or "data requirements," but they should not be rejected out of hand in such instances since one or more other criteria (e.g., "availability of application software") may mitigate a poor rating on "ease of use" or "data requirements." Per the taxonomy set out in Appendix B, the conjunctive and dominance techniques are relatively easy to implement, requiring no information on the relative importance of attributes. AHP, MAVT, and MAUT, on the other hand, require more information on the relative importance of attributes and are computationally more involved. They are nonetheless readily applicable to the typical COEA problem, and may be described as "practical." The assessment level of the techniques ranges from ordinal (dominance) to stochastic (MAUT).

EEA 3. How do the most practical techniques compare in a typical COEA application?

In connection with the Tow Sight Improvement Program (TSIP) COEA, we make the assumption that dominance or the conjunctive technique may well have been applied early in the process of delimiting alternatives. Thus, these techniques may be viewed as "preparatory" tools to be used early in the analytic process whereas AHP, MAVT, or MAUT might well be viewed as "finishing" machinery in that process.

AHP and MAUT yield similar demonstration results; that is, each, under "neutral" circumstances, highlights the Base Case as the apparent choice for a decision maker. Each technique also shows Alternative 3, under similar circumstances, to be a "close second." Moreover, each technique allows sensitivity analysis such that an analyst may demonstrate to a decision maker the impact of differing attribute emphases, or values, on the outcomes. The

decision maker may become heavily involved in the analytic process, for either MAUT or AHP, as the analyst requires serious contemplation of attributes.

AHP and MAUT differ in several respects. Per the taxonomy in the Compendium, AHP involves a lower level of assessment as well as a somewhat less burdensome requirement in connection with attribute information. Conceptually, this probably makes it somewhat more accessible than MAUT, if not more readily carried out. MAUT holds more appeal for those analysts who prefer a less "plodding," more "elegant" solution. Both techniques, however, now enjoy the advantage of generally available software to ease computational pain on the part of analysts. Practically speaking, the level of information available would reasonably drive the choice of one technique or another in a given COEA.

b. Conclusions.

(1) A number of multiattribute integration techniques, occupying a range of niches in a taxonomy, show promise for use in COEA. Five of these (refer to EEA 2) may currently be described as "practical." Others may prove useful in light of future research.

(2) Several techniques (e.g., AHP, MAUT) permit integration of attribute estimates or outcomes into an overall scheme to guide a decision maker in the choice of alternatives.

(3) Techniques such as those demonstrated may be applied - with the help of generally available software - to COEA. Such application will help minimize present objectionable analytical peculiarities and variances.

Appendix A

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Appendix B

Compendium of Techniques

1. Introduction. This annotated compendium of techniques was assembled in accord with the scope and limitations of the study. It includes (1) a brief discussion of categories of assessment and types of information on attributes, (2) a table summary of techniques, and (3) an explication of those techniques. Primary references are Chankong and Haimes (1983), Hwang and Yoon (1981), Keeney and Raiffa (1976), and MacCrimmon (1973).

2. Assessment Categories. The COEA alternatives are assessed with regard to each attribute. Assessments may use combat simulation models, field tests, historical databases, expert opinion, or other analytic methods. For present purposes, the study team assumes that attributes are monotonic in the sense that either more is always better, or less is always better. For the purpose of this compendium the type of assessment is categorized as ordinal(rank), cardinal(numeric), or stochastic.

a. Ordinal Assessment. In this case only a ranking of alternatives is provided. For example, suppose three alternatives A, B, and C are to be compared with regard to the attribute, total comparative life cycle cost. An ordinal assessment may yield only that the cost of alternative B is greater than the cost of alternative C is greater than the cost of alternative A.

b. Cardinal(numeric). In this case numeric outcomes are provided. Comparing the three alternatives with regard to total comparative life cycle cost, a cardinal assessment may yield \$490 million for the cost of alternative A, \$850 million for the cost of alternative B, and \$640 million for the cost of alternative C.

c. Stochastic. In this case a probability distribution is provided over the attributes for each alternative. Continuing the example, a stochastic assessment may yield that the cost of alternative A is uniformly distributed from \$440 million to \$540 million, that the cost of alternative B is uniformly distributed from \$800 million to \$900 million, and that the cost of alternative C is uniformly distributed from \$590 million to \$690 million.

d. The progression of assessment types: ordinal, cardinal, stochastic, constitutes a progression of levels of measurement and generally a more complex assessment process.

3. Information Types. In general the techniques of the compendium require information on relative importance attached to the attributes by the decision maker. For the

purpose of this compendium the type of information on relative importance is categorized as none, ordinal, simple weighting, and tradeoff.

a. No information. The decision maker is required to provide no information on the relative importance of the attributes. Techniques for this type are applicable regardless of the relative importance attached to the attributes by the decision maker.

b. Ordinal information. The decision maker is required to rank order the attributes in importance. For example, suppose a decision problem has the following attributes: X, total comparative life cycle cost, Y, loss exchange ratio in defense, and Z, loss exchange ratio in the offense. Ordinal information from the decision maker may yield that Z is more important than Y is more important than X.

c. Simple weighting information. The decision maker is required to provide information from which numerical weights for each attribute can be derived. These weights capture the relative importance of the attributes. Continuing the example, the decision maker may provide information which leads to the weights: 0.23 for total comparative life cycle cost, 0.47 for loss exchange ratio in the defense, and 0.72 for loss exchange ratio in the offense. In order for the decision maker to provide meaningful weights, the attributional scales must have some degree of comparability. If the original assessment scales do not possess this comparability, new scales may be developed and corresponding outcomes assessed by the decision maker.

d. Tradeoff information. The decision maker is required to provide enough information to develop a value or utility function over the complete set of attributes. The decision maker most often provides this information in the form of tradeoffs. For example, given a cost of \$450 million, a loss exchange ratio in defense of 2.0, and a loss exchange ratio in offense of 0.5; what increase in cost would be traded off for an increase to 1.0 for the loss exchange ratio in the offense?

e. The progression of types of information: none, ordinal, simple weighting, tradeoff, constitutes a progression of levels of measurement which becomes increasingly burdensome to the decision maker.

4. Using the three types of assessment and the four types of information on the relative importance of attributes, we suggest a taxonomy to categorize the techniques of this compendium. Table 1 illustrates the techniques of the compendium in the context of this taxonomy.

Table 1
Multiattribute Techniques

INFORMATION ON RELATIVE IMPORTANCE OF ATTRIBUTES	ASSESSMENT		
	Ordinal	Cardinal	Stochastic
None	Dominance Maximin Maximax Majority Rule Koler's	Conjunctive Disjunctive Cost-Effect ratio(value)	Stochastic dominance Cost-Effect ratio(utility)
Ordinal	Lexicographic	Lexicographic with minima Key attribute	
Simple Weighting	ELECTRE Permutation	AHP SAW TOPSIS	
Tradeoff		Value Theory (MAVT)	Utility Theory (MAUT)

Explications of the techniques of the table follow.

a. Dominance. Alternative A dominates alternative B if it is better on at least one attribute and at least as good on all others. Consider the following two attribute decision. Alternative A has cost \$500 million and loss

exchange ratio 1.7. Alternative B has cost \$350 million and loss exchange ratio also 1.7. Alternative A then dominates alternative B. This is a practical technique because the attributes are assumed to be monotonic (more is always better, or less is always better). Of all the techniques this is the simplest and most obvious.

b. Maximin. In this technique each alternative is represented by its worst (minimum) outcome on any of the attributes. The technique then selects the alternative with the best (maximum) outcome. A high degree of comparability must exist among the attributional scales. If this is not the case new scales may be developed. The decision maker may then reassess the alternatives using these new scales.

c. Maximax. In this technique each alternative is represented by its best (maximum) outcome on any of the attributes. The technique then selects the alternative with the best (maximum) outcome. As with the previous technique a high degree of comparability must exist among the attributional scales. As before if this is not the case new scales may be developed, and the decision maker may then reassess the alternatives using these new scales.

d. Majority rule. Alternative A ranks before alternative B if assessments on a majority of the attributes rank alternative A before alternative B. This method is applicable if the decision maker ranks the attributes equal in relative importance. Although conceptually very simple, this technique can lead to an overall ranking which is not transitive. Consider the following decision situation:

Attributes	Rankings of Alternatives
X	A > B > C
Y	B > C > A
Z	A > C > B

Majority rule yields an overall rank of A > B > C. If the ranking on attribute Z were changed to C > A > B, the overall ranking would be A > B, B > C, but C > A. Although people certainly exhibit decision behavior which is not always transitive, for prescriptive purposes techniques which may lead to intransitive results are not recommended.

e. Koler's technique. Arrow & Raynaud (1986) investigated ranking techniques that generalize the majority rule. Their concern was timely business decisions that have perhaps hundreds of alternatives and hundreds of attributes. Ranking alternatives in this situation is termed the industrial outranking problem. The technique is illustrated

in the following situation. There are four alternatives numbered 1 through 4, which are compared on the basis of seven attributes denoted A through G.

Attributes	Rankings of Alternatives
A	3 > 2 > 4 > 1
B	2 > 1 > 4 > 3
C	1 > 3 > 2 > 4
D	4 > 2 > 1 > 3
E	4 > 3 > 1 > 2
F	4 > 1 > 3 > 2
G	1 > 2 > 3 > 4

The technique is based on the outranking matrix. In the i th row and j th column of this matrix is the number of attributes that rank alternative i before alternative j . The technique assumes that the intensity of preference between alternatives i and j is a strictly increasing function of the number of attributes that rank i before j . The outranking matrix for the above data is

*	4	5	3
3	*	3	4
2	4	*	3
4	3	4	*

The technique proceeds stepwise to identify what are defined to be the prudent rankings. These are transitive rankings with pairwise comparisons holding for the largest possible number of attributes. First, the minimum entry of each row is identified. Next, the maximum of these minima is chosen with ties broken arbitrarily. The row index of the maximum indexes the alternative ranked at this step. The row and column with this index are deleted from the matrix, and the process is repeated until the matrix is empty. The above matrix yields the rankings: 1243, 1324, 1432, 2413, and 4132.

As the example illustrates the technique may not produce a unique ranking. Although the technique does not identify all prudent rankings, if there is a unique prudent ranking the technique will produce it. Also if the majority rule technique would yield a transitive ranking, Koler's technique will produce it as the unique prudent ranking. Because of its simplicity and ease of use, Koler's technique has potential in COEA application. However, further

research is required for a full understanding for its application in a COEA environment.

f. Conjunctive technique. This technique requires that minimum acceptable levels be established for each attribute. Alternatives are then categorized as acceptable or unacceptable according to whether they meet or better **all** the minima or not. Like dominance this is a simple and obvious technique. For use with COEA establishing the minimum levels may present difficulties. Minima for some attributes may be available from an Operational Requirements Document(ORD).

g. Disjunctive technique. This technique requires that exceptional levels be established for each attribute. Alternatives are then categorized as acceptable or unacceptable according to whether or not they meet or better **at least one** of the exceptional levels. As with the conjunctive technique determination of the levels limits the practical application of this technique. Exceptional levels may not be readily established in most COEA applications.

h. Stochastic dominance. This technique is the stochastic analogue of the previous dominance technique. Assume assessments have resulted in probability distributions, one for each alternative, over the attributes. Alternative A stochastically dominates alternative B, if the outcomes of A are better than those of B with probability one. When the individual attributes have independent probability distributions this technique is easily applied.

i. Lexicographic technique. This is the ordering technique of a dictionary. Alternatives are first ranked according to the most important attribute. If two alternatives tie on this attribute, they are then ranked according to the second most important attribute. If they tie according to this attribute the third most important attribute is used, and so on. Consider the decision situation with alternatives A, B, and C and attributes X, Y, and Z where the order of the attributes in relative importance is $X > Y > Z$.

Rankings of Alternatives	
Attributes	
X	$A > B = C$
Y	$B = A > C$
Z	$C > B > A$

Lexicographic technique yields an overall ranking of $A > B > C$.

In COEA the lexicographic technique might be more useful if a practical level of difference were established for each attribute. Two alternatives would be ranked equal on an attribute if their outcomes were within the corresponding practical level of difference. Practical levels of difference would take into account the accuracy of the corresponding assessment technique. For example, a practical level of difference for a cost attribute might be determined to be \$10 million. Alternative A with a cost of \$523 million and alternative B with a cost of \$517 million would then be ranked equal on the cost attribute, for application of this technique.

j. Lexicographic with minima. As in the conjunctive technique minimum acceptable levels must be established for each attribute. Alternatives are first ranked according to the most important attribute, if its minimum level is not met. If the minimum level for the most important attribute is met but the minimum level for the second most important attribute is not, the alternatives are ranked according to the second most important attribute. If the minimum level of the second most important attribute is met but the minimum level of the third most important attribute is not met, alternatives are ranked by the third most important attribute, and so on. This technique is similar to the more complex Elimination by Aspects (EBA).

k. Key attribute. Minimum acceptable levels are established for all attributes, except one, the key attribute. Alternatives which do not meet or better all minima are unacceptable. Acceptable alternatives are then ranked according to the key attribute. In use with COEA minimum levels might be established for all attributes except cost, with cost identified as the key attribute. Alternatives which do not meet the minimum levels are eliminated as unacceptable. The acceptable alternatives are then ranked by cost.

l. Permutation. This technique successively computes an index value for each possible ranking(permutation) of alternatives. The ranking with the highest index is recommended. Consider the decision situation with alternatives A and B and attributes X and Y.

Rankings of Alternatives

Attributes and Weight		
X	0.4	A > B
Y	0.6	A < B

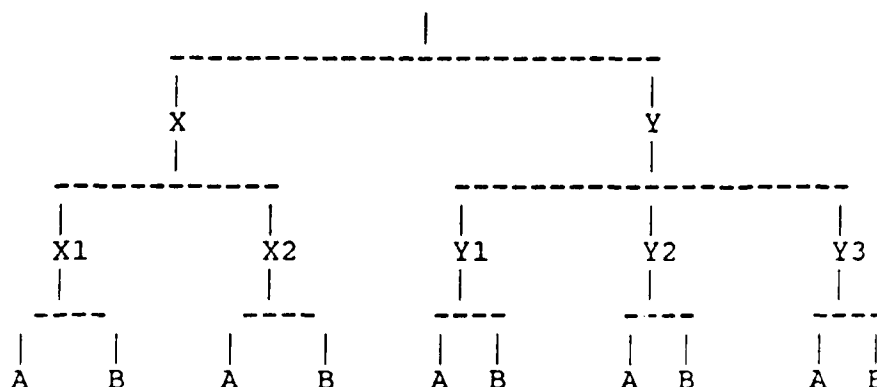
X	0.4	A > B
Y	0.6	A < B

Consider the overall ranking(permutation) of A > B. Since A > B on attribute X, attribute X constitutes the set of concordance. Since A < B on attribute Y, attribute Y constitutes the set of discordance. The index for the

overall ranking is the weight of attribute X(concordance) minus the weight of attribute Y(discordance), -0.2 . Likewise, consider the overall ranking(permutation) of $A < B$. Since $A > B$ on attribute X, attribute X constitutes the set of discordance. Since $A < B$ on attribute Y, attribute Y constitutes the set of concordance. The index for the overall ranking is the weight of attribute Y (concordance) minus the weight of attribute X(discordance), 0.2 . Since the overall ranking(permutation) $A < B$ has the highest index it is recommended.

m. ELECTRE. This technique (Elimination et Choix Translation Realite) is similar to the permutation technique in that indices involving concordances and discordances are computed. However, indices are only computed for pairs of alternatives. Alternatives are then ordered based on these indices. Unlike the permutation method ELECTRE requires the decision maker to set arbitrary threshold values for the indices. The resulting ranking is in general neither complete nor transitive. The computations required in the permutation technique or ELECTRE could easily be implemented in commercially available spreadsheets.

n. AHP. The Analytic Hierarchy Process(AHP) is described in Saaty(1980). Its axiomatic foundations are given in Saaty(1989). The two key features of AHP are a hierarchy of attributes and the eigenvector weighting method. The weighting method typically uses as scale values the integers 1 to 9 and their reciprocals. Consider the decision situation with alternatives A and B and a two level hierarchy of attributes. Attribute X is cost, and attribute Y is effectiveness. Subattribute X1 is peace time cost and subattribute X2 is war time cost. Subattributes Y1, Y2, and Y3 are effectiveness under scenarios one, two, and three, respectively.



This hierarchy requires eight distinct scalings. Alternatives are pairwise compared for their contribution to each of the five lowest level attributes. Next, the subattributes X1 and X2 are compared for the importance of their contribution to the attribute X, and the attributes

Y1, Y2, and Y3 are compared for their contribution to the attribute Y. Finally, the attributes X and Y are compared for their importance to the overall decision.

Table 2 illustrates the scale used to assess the pairwise importance of the attributes or alternatives in the hierarchy.

Table 2
AHP Scale of
Pairwise Importance

Importance of one factor over another	Value
Equal	1
	2
Weak	3
	4
Strong	5
	6
Very Strong	7
	8
Absolute	9

The AHP importance scale also makes use of the reciprocals of the values in Table 2. For example, if the pairwise importance of factor 1 over factor 2 is judged to be weak and valued at 3, the pairwise importance of factor 2 over factor 1 is valued at 1/3.

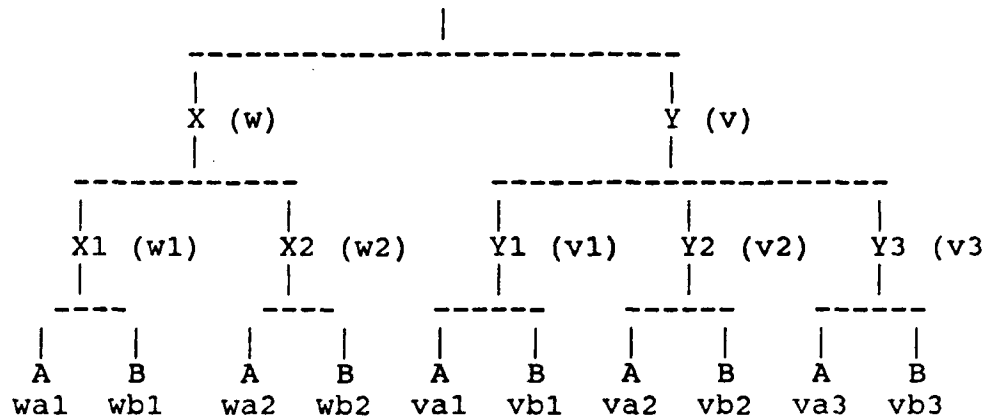
Pairwise importances for a set of attributes or alternative are assessed in a matrix format. For example, consider the relative assessment of the three attributes: Y1, Y2, Y3. Suppose that Y1 is judged to be strongly more important than Y2 and absolute more important than Y3. Suppose also that Y2 is judged to be weakly more important than Y3. Table 3 illustrates these scale values in matrix format.

Table 3
Pairwise Comparisons of
Attributes Y1, Y2, Y3

	Y1	Y2	Y3
Y1	1	5	9
Y2	1/5	1	3
Y3	1/9	1/3	1

The principal eigenvalue or largest real eigenvalue of this matrix is 3.029. Its associated eigenvector, normalized so its components sum to one, is (0.7514, 0.1782, 0.0704). The eigenvector weighting method assigns Y1 a weight of 0.7514, Y2 a weight of 0.1782, and Y3 a weight of 0.0704. Note that the assessed values of Table 3 are not completely consistent. If the 5 entry and 1/5 entry were changed to 3 and 1/3, respectively, the scaling becomes completely consistent with a principal eigenvalue of 3. In general the principal eigenvalue is at least as large as the dimension of the matrix, in this example, 3. The greater the eigenvalue the more inconsistent the data.

Once all weights are computed by the eigenvector method an overall weight for each alternative is computed. Weights for each alternative are computed linearly up the hierarchy. Continuing the example suppose the assessments and eigenvector method lead to the following weights, symbolized with lower case letters.



The overall weight of alternative A is

$$w*(w1*wa1 + w2*wa2) + v*(v1*va1 + v2*va2 + v3*va3)$$

The overall weight of alternative B is

$$w*(w1*wb1 + w2*wb2) + v*(v1*vb1 + v2*vb2 + v3*vb3)$$

Alternatives are now ranked by overall weight. Although the decision maker is required to make a number of assessments, the nine point scale simplifies the required scaling. Commercial personal computer software is available to implement AHP. The only difficult computations are those to determine the eigenvalues and eigenvectors. Most numerical analysis software libraries have routines to perform these computations. Eigenvalue and eigenvector computations for this study were accomplished using IMSL, a commercially available FORTRAN library. Computation of overall weights for alternatives could easily be implemented in commercially available spreadsheets.

o. Value Theory. Multiattribute value theory(MAVT) encompasses a number of techniques that have a common theoretical basis. This basis is measurement theoretic and relies on the concept of a preference or value function. A preference function is intended to measure a decision maker's preference for various combinations of outcomes on the attributes. Consider an example with alternatives, A and B, and attributes, $X_1, X_2, X_3, \dots, X_n$. Alternative A has outcomes $x_{1a}, x_{2a}, x_{3a}, \dots, x_{na}$, and alternative B has outcomes $x_{1b}, x_{2b}, x_{3b}, \dots, x_{nb}$. If the decision maker's preference over the attributes is represented by a preference function F , alternative A is ranked before alternative B if

$$F(x_{1a}, x_{2a}, x_{3a}, \dots, x_{na}) > F(x_{1b}, x_{2b}, x_{3b}, \dots, x_{nb})$$

Typically value is only an ordinal concept hence the function F is only unique up to a strictly monotonically increasing transformation. See Roberts(1979) for the theory. Commercial personal computer software is available to implement multiattribute value theory.

(1) Indifference. The concept of indifference provides a technique for investigating preference functions. An indifference set consists of all combinations of outcomes which are equally valued by the preference function. In two dimensions the indifference set is simply a curve. For example, cost and effectiveness outcomes (C_a, E_a) and (C_b, E_b) are on the same indifference curve of the preference function F if

$$F(C_a, E_a) = F(C_b, E_b)$$

This means that the decision maker is indifferent with regard to alternative A (with cost C_a and effectiveness E_a) and alternative B (with cost C_b and effectiveness E_b). Algebraically an indifference curve is defined by the relationship:

$$F(\text{Cost}, \text{Effectiveness}) = \text{constant}$$

Graphically an indifference curve may be represented in the cost-effectiveness plane as shown in Figure 1 below.

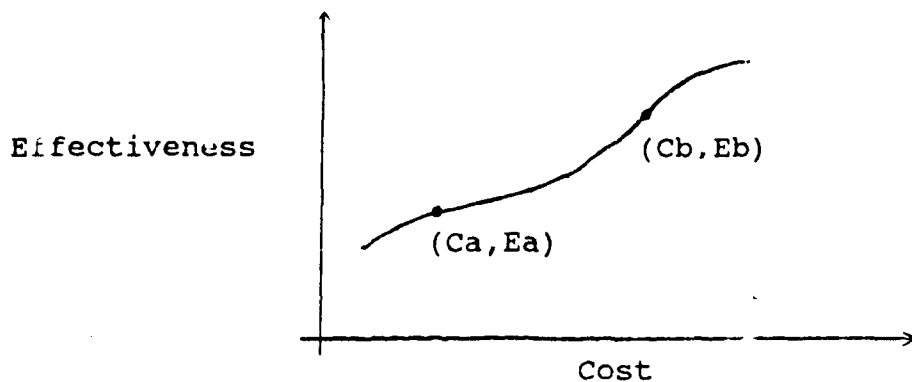


Figure 1. An Indifference Curve

All combinations of cost and effectiveness along the curve of Figure 1 are equally preferred.

(2) Some properties of preference functions. In general preference relationships will vary by individual decision makers. Consequently there is no one correct preference function for ranking alternatives. However, there may be reasonable behavioral rationale for limiting the types of preference functions and thus constraining the shapes of the indifference sets. Some properties of preference functions are illustrated assuming only two attributes, cost and effectiveness.

It is likely that a decision maker prefers alternative A to alternative B if alternative A is at least as effective but less costly or more effective but no more costly. If this is the case the decision maker's preference relationship exhibits the property of dominance. This property is simply the dominance technique previously discussed now in the context of a preference function. Dominance requires that the preference function be increasing with increasing effectiveness or decreasing cost.

The following two typical preference properties (PP) are discussed in detail in Keeney and Raiffa (1976)

PP1: At a fixed level of cost as effectiveness increases the decision maker will pay less for a fixed increase in effectiveness

and

PP2: At a fixed level of effectiveness as cost increases the decision maker will pay less for a fixed increase in effectiveness

These two properties are consistent with the concave upward indifference curves illustrated below:

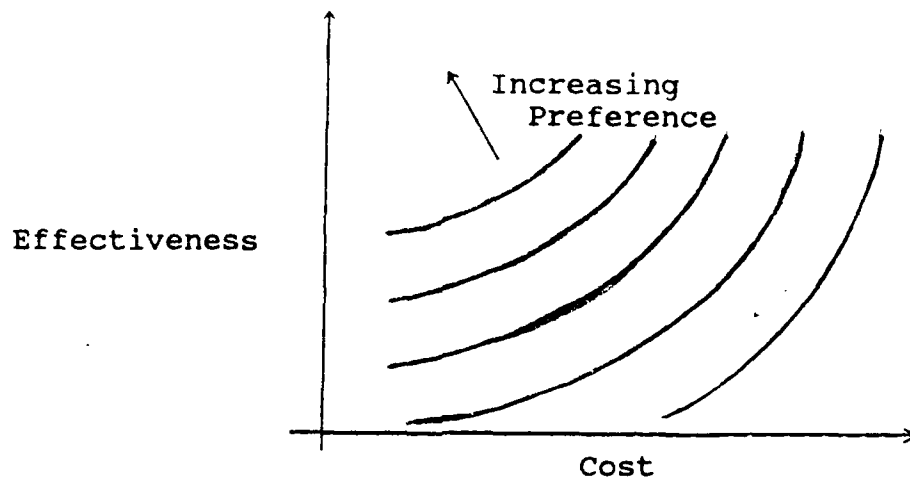


Figure 2. Indifference Curve with PP1 and PP2

The indifference curves of Figure 2 may be contrasted with those of Figure 3 below.

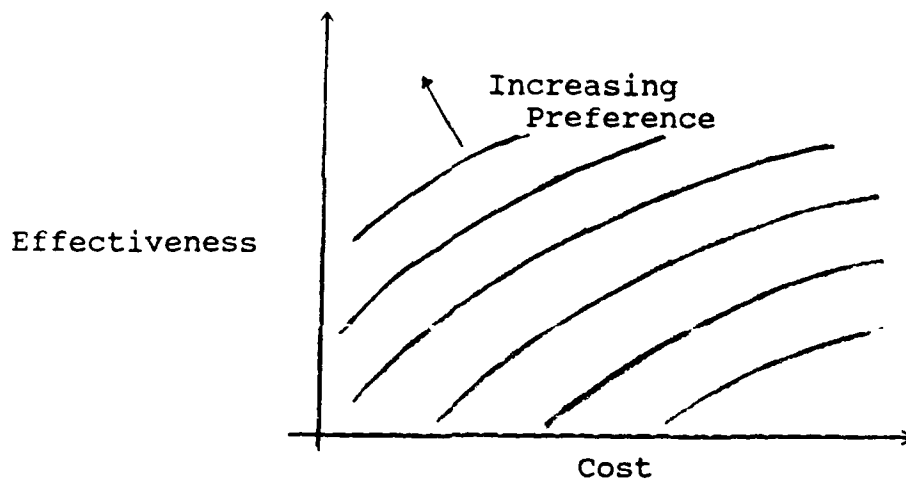


Figure 3. Indifference Curve with PP3 and PP4

The concave downward curves of Figure 3 are found when the following two preference properties hold

PP3: At a fixed level of cost as effectiveness increases the decision maker will pay **more** for a fixed increase in effectiveness

and

PP4: At a fixed level of effectiveness as cost increases the decision maker will pay **more** for a fixed increase in effectiveness

For more on shapes of indifference curves see MacCrimmon and Wehrung(1977).

A simpler type of indifference curve occurs when the decision maker will pay the same fixed increase in cost for the same fixed increase in effectiveness anywhere in the cost-effectiveness plane. Equivalently, the rate of substitution of effectiveness for cost is constant. In this case preference can be represented by a linear function of cost and effectiveness and the indifference curves form a family of parallel lines as illustrated in Figure 4. The slope of the parallel lines equals the constant rate of substitution of effectiveness for cost.

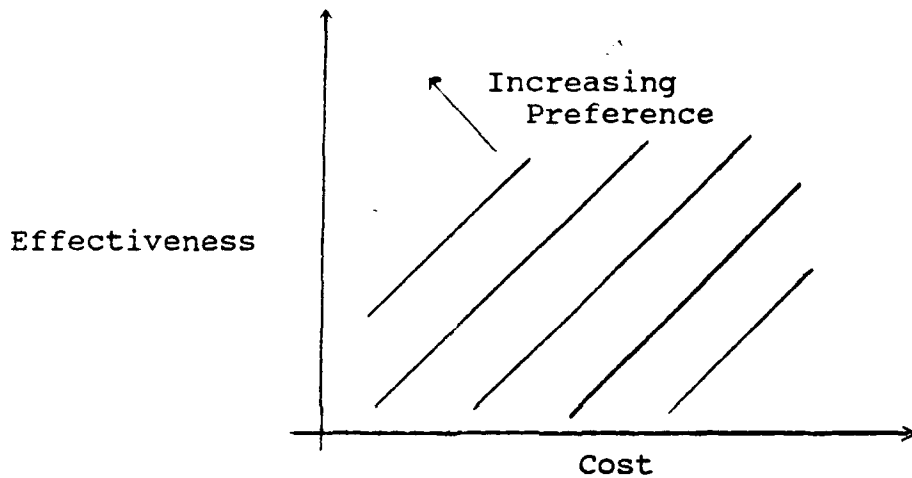


Figure 4. Linear Indifference Curves

In this linear case alternatives can simply be compared by the ratio of their difference in effectiveness to their difference in cost. Let R be the constant rate of substitution of effectiveness for cost. For alternatives A (C_a, E_a) and B (C_b, E_b) with B the more effective let

$$S = (E_b - E_a) / C_b - C_a$$

Then A is preferred if $S < R$, B is preferred if $S > R$, and A and B are equally preferred if $S = R$.

(3) Additive Value. In general, determination of a decision maker's preference function is a complex task. Keeney and Raiffa(1976) describe the details of the process. Simplification occurs when linearity properties hold. In this situation the decision maker's preferences may be represented by a function F that decomposes as

$$F(X_1, X_2, X_3, \dots, X_n) = F_1(X_1) + F_2(X_2) + F_3(X_3) + \dots + F_n(X_n)$$

a sum of the single attribute preference functions $F_1, F_2, F_3, \dots, F_n$. Determination of the single attribute functions is in theory a simpler task than determining a multiattribute function. The two attribute example with a constant rate of substitution of cost for effectiveness is a special case of such a linear decomposition. An alternate form has the function decomposing as

$$F(X_1, X_2, X_3, \dots, X_n) =$$

$$w_1 * f_1(X_1) + w_2 * f_2(X_2) + w_3 * f_3(X_3) + \dots + w_n * f_n(X_n)$$

and

$$w_1 + w_2 + w_3 + \dots + w_n = 1$$

A number of techniques simply assume a linear decomposition of the preference function. A justification for this assumption is that additive value yields a robust approximation. See Dawes and Corrigan(1974).

p. SAW. The technique of Simple Additive Weighting(SAW) is a common additive value technique. This technique assumes a linear decomposition of the preference function. However, rather than determining the single attribute preference functions explicitly, the technique relies on comparable preference scales for the attributes and weights for their relative importance. For example, suppose the assessed outcomes for alternative A are $x_1, x_2, x_3, \dots, x_n$. The decision maker converts these to comparably scaled preference outcomes $y_1, y_2, y_3, \dots, y_n$. The decision maker also provides relative weights; $w_1, w_2, w_3, \dots, w_n$ for the attributes. The preference function is then evaluated for alternative A by:

$$F(A) = w_1 * y_1 + w_2 * y_2 + w_3 * y_3 + \dots + w_n * y_n$$

Although differing in theory, the form here is very similar to that of AHP . The application may be even more similar if the eigenvector weighting process is used to determine the scaled outcomes y_i 's and the attribute weights w_i 's.

q. Cost-effectiveness ratios. If there is only a cost attribute and an effectiveness attribute, alternatives could be ranked by their cost-effectiveness ratios. From the standpoint of value theory, ranking by cost-effectiveness ratios assumes a ratio preference function F as follows

$$F(\text{Cost}, \text{Effectiveness}) = \text{Effectiveness} / \text{Cost}$$

Alternative A, with cost C_a and effectiveness E_a is ranked before alternative B, with cost C_b and effectiveness E_b if

$$F(C_a, E_a) = E_a / C_a > F(C_b, E_b) = E_b / C_b$$

Preference that is represented by the ratio preference function also has an additive representation. Since the logarithmic transformation is strictly monotonically increasing the functions

$$\begin{aligned} & \text{Effectiveness / Cost} \\ \text{and} \\ & \log(\text{Effectiveness / Cost}) \end{aligned}$$

yield the same preference ordering. However the logarithmic form decomposes as

$$\begin{aligned} \log(\text{Effectiveness / Cost}) &= \log(\text{Effectiveness}) \\ &+ \log(1 / \text{Cost}) \end{aligned}$$

The cost-effectiveness ratio is an example of a derived measure, wherein fundamental measures, cost and effectiveness, are combined mathematically to produce a new or derived measure. A standard example of a derived measure is density, the ratio of the fundamental measures mass and volume. The same fundamental measures can be combined in different ways to produce distinct derived measures. For example, the length, width, and height of rectangular boxes may be combined to produce either the surface area measure or the volume measure. Care should be taken in assuming that the mathematics of any particular derivation consistently reflects a decision maker's preference relationships over the fundamental measures. Ranking by cost-effectiveness ratios makes this assumption.

Indifference curves of the ratio preference function are rays emanating from the origin as shown in Figure 5.

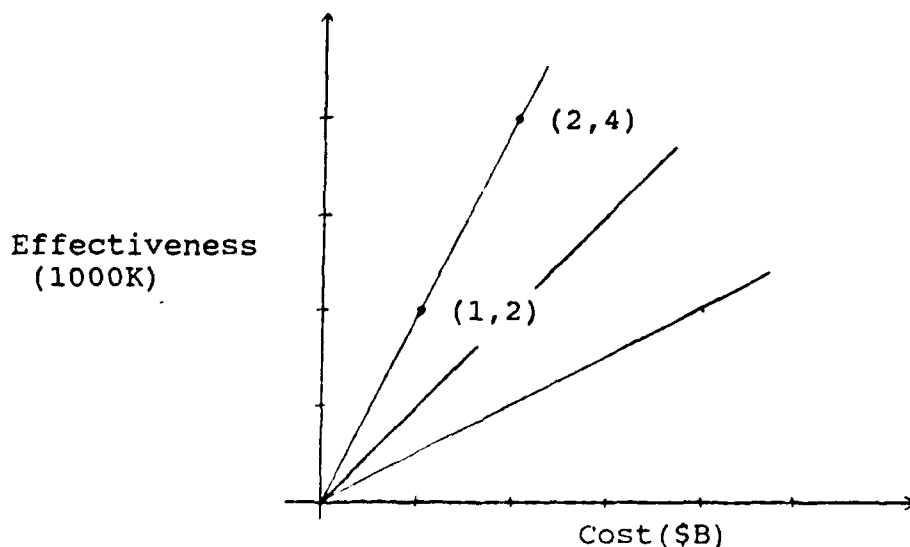


Figure 5. Indifference Curves of the Ratio Preference Function

As shown in the figure cost and effectiveness combinations (1.0 billion dollars and 2000 kills) and (2.0 billion dollars and 4000 kills) lie on the same indifference curve. Equivalently, the decision maker is indifferent with regard to the alternative with cost 1.0 billion dollars and effectiveness 2000 kills and the alternative with cost 2.0 billion dollars and effectiveness 4000 kills. Any other combination of cost and effectiveness with the same kill to dollar ratio would lie on this indifference curve.

Preference property one (PP1) holds for the ratio preference function. However, this function also exhibits preference property four (PP4). This fact is illustrated in the next figure.

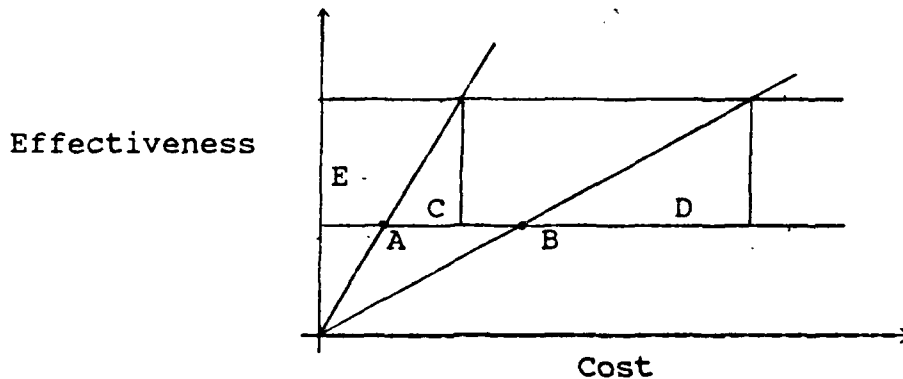


Figure 6. Amounts Paid for Fixed Increase in Effectiveness

In Figure 6 alternatives A and B have the same effectiveness. Alternative B has the greater cost. An increase of E in effectiveness is worth an increase of C in cost at A and an increase of D in cost at B. It can be seen from the figure that D is greater than C. Thus more is being paid for a fixed increase in effectiveness at a higher level of cost.

The advantage of cost-effectiveness ratios is that the decision maker is required to provide little or no information. The disadvantage is that the decision maker must accept the unique properties which this technique requires.

r. TOPSIS. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) develops a preference function based on distances between alternatives as represented in attribute space. This technique requires only that the decision maker give relative weights for the attributes. These weights are normalized and then combined with normalized outcomes on the attributes to form the decision outcomes. The best decision outcomes become the outcomes of the ideal alternative. The worse decision

outcomes become the outcomes of the negative ideal alternative. Alternatives are then ranked using their distances from these two hypothetical alternatives.

Consider an example with alternatives A, B, and X and two attributes, cost and effectiveness. The situation is illustrated in Figure 7 below. Alternatives are represented graphically by their normalized weighted outcomes (C_a, E_a) , (C_b, E_b) , and (C_x, E_x) . The ideal alternative I has the best cost, C_b , and the best effectiveness, E_a . The negative ideal alternative N has the worst cost, C_a , and the worst effectiveness, E_b .

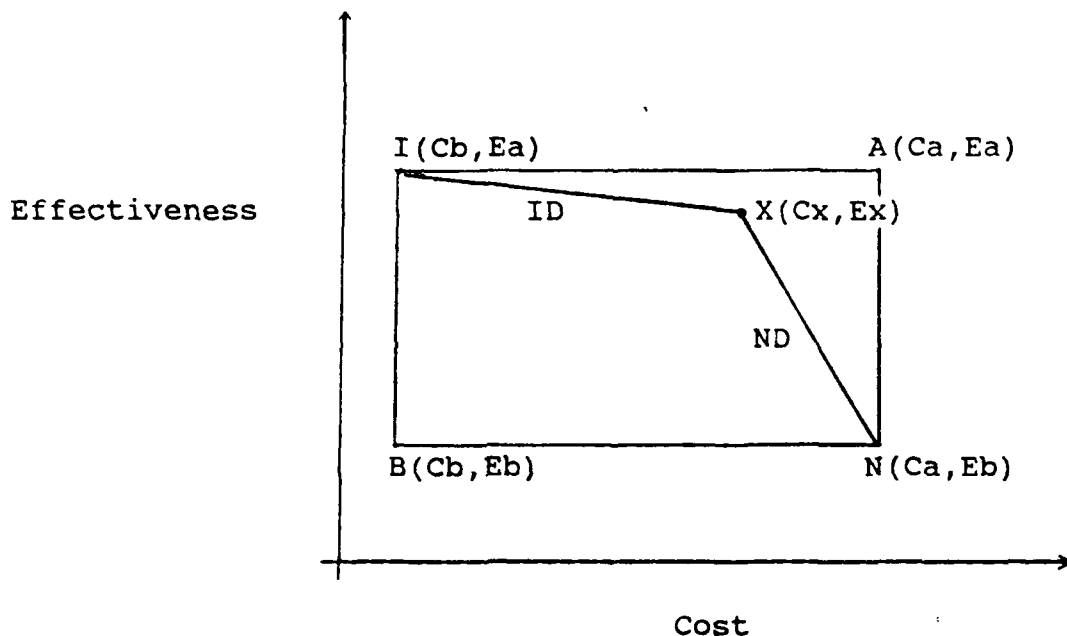


Figure 7. Computing the TOPSIS Index

For alternative X the distance, ID, between it and the ideal alternative and the distance, ND, between it and the negative ideal alternative are computed. The index of alternative X is defined to be

$$I(X) = ND / (ID + ND)$$

The indices of the other alternatives are computed similarly. Alternatives are then ranked according to their indices.

TOPSIS indifference curves, assuming that cost and effectiveness outcomes have been normalized and weighted, are illustrated in Figure 8 below. One indifference curve is the perpendicular bisector, L, of the line segment joining the ideal and negative ideal alternatives. Other indifference curves are segments of circles (Circles of Apollonius). Segments above L are concave upward and

segments below L are concave downward. Preference properties PP1 and PP2 hold above L. Preference properties PP3 and PP4 hold below L. This situation should not be assumed to hold for a decision maker's preference without a more detailed examination.

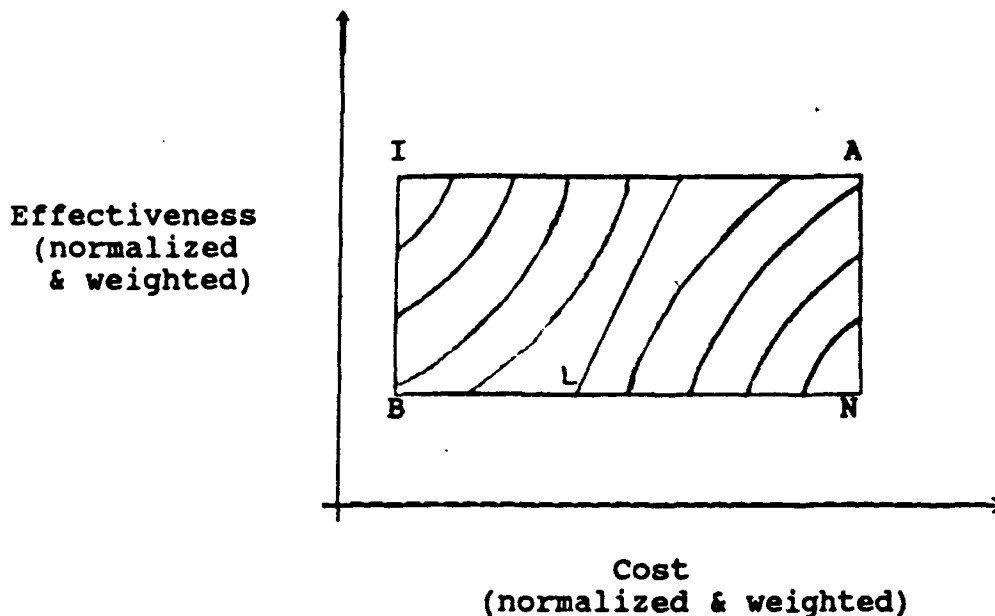


Figure 8. TOPSIS Indifference curves

The next example illustrates that TOPSIS does not satisfy the property of independence of irrelevant alternatives (see Luce and Raiffa, 1957). Assume that the decision maker equally weights cost and effectiveness. Table 2 gives outcomes (suitably scaled) for the cost and effectiveness of four alternatives, A through D. It also gives the TOPSIS index.

Table 2
Initial TOPSIS Results

Alternative	Cost	Effectiveness	TOPSIS Index
A	4.0	4.0	0.624
B	4.1	10.0	0.874
C	6.0	12.0	0.860
D	16.0	12.1	0.376

Alternative B is ranked first, C second, A third, and D last. Suppose new data or further analysis result in a change in the effectiveness of alternative D from 12.1 to 13.1. The revised data together with the new indices are given in the Table 4.

Table 4
Updated TOPSIS Results

Alternative	Cost	Effectiveness	TOPSIS Index
A	4.0	4.0	0.603
B	4.1	10.0	0.828
C	6.0	12.0	0.846
D	16.0	13.1	0.397

The rank of alternative D is not changed, it is still a distant fourth. However, alternative C is now ranked first and B second, reversing the previous pairwise ranking. Thus the ranking of the two alternatives has switched because of a change to the outcome of a poorly ranked, or irrelevant, alternative.

An advantage of TOPSIS is that it requires little information from the decision maker. Disadvantages are that it requires the acceptance of a unique set of indifference curves and the rejection of the property of independence of irrelevant alternatives.

The next example illustrates that TOPSIS and cost-effectiveness ratios may yield different results. Table 5 gives cost and effectiveness outcomes for three alternatives. In addition the TOPSIS index and effectiveness to cost ratio are given.

Table 5
TOPSIS and Cost-effectiveness Ratios

Alternative	Cost	Effectiveness	Eff/Cost	TOPSIS Index
A	1.0	4.0	4.0	0.719
B	2.0	7.0	3.5	0.877
C	12.0	8.0	0.667	0.281

Using the effectiveness to cost ratios, alternative A ranks first and alternative B second. However, using the TOPSIS index, alternative B ranks first and alternative A second.

s. **Utility Theory.** Multiattribute utility theory is a stochastic analogue of value theory. The value or preference function is replaced by the utility function. Outcomes of the alternatives are assessed with probability distributions rather than with point estimates. The theory and application are described in Keeney and Raiffa(1976).

This technique is subsumed by the method of decision analysis. A basic axiom of decision analysis is maximization of expected utility. Consider an example with

alternatives, A and B, and attributes, $x_1, x_2, x_3, \dots, x_n$. Alternative A has a probability distribution of outcomes

$$f_a(x_1, x_2, x_3, \dots, x_n)$$

and alternative B has a probability distribution of outcomes

$$f_b(x_1, x_2, x_3, \dots, x_n)$$

If the decision maker's preference over the attributes is represented by utility function U , the expected utility of alternative A is

$$\int U(x_1, x_2, x_3, \dots, x_n) * f_a(x_1, x_2, x_3, \dots, x_n)$$

and the expected utility of alternative B is

$$\int U(x_1, x_2, x_3, \dots, x_n) * f_b(x_1, x_2, x_3, \dots, x_n)$$

Alternative are then ranked by their expected utility.

One of the strengths of MAUT is that in the context of decision analysis it accommodates a coherent integration with a theory of probability and statistical inference. This allows for a rigorous mathematical treatment of uncertainty and risk.

(1) Additive Utility. Determination of a decision maker's utility function is a highly complex task as Keeney and Raiffa(1976) show. The process greatly simplifies if a property of additive independence holds. In this situation the decision maker's utility may be represented by a function U that decomposes as

$$U(x_1, x_2, x_3, \dots, x_n) =$$

$$w_1 * u_1(x_1) + w_2 * u_2(x_2) + w_3 * u_3(x_3) + \dots + w_n * u_n(x_n)$$

a sum of the single attribute utility functions $u_1, u_2, u_3, \dots, u_n$. Determination of the single attribute functions is in theory a simpler task than determining a multiattribute function.

(2) Multilinear Utility. A somewhat more complex decomposition results when a property known as utility independence holds. See Theorem 6.3 of Keeney and Raiffa. In this case the decision maker's utility may be represented by a function U that decomposes as

$$\begin{aligned} U(x_1, x_2, x_3, \dots, x_n) = & \sum w_i * u_i(x_i) \\ & + \sum w_{ij} * u_i(x_i) * u_j(x_j) \\ & + \dots \\ & + \sum w_{ijn} * u_i(x_i) * u_j(x_j) * \dots * u_n(x_n) \end{aligned}$$

where as with the additive decomposition the u_i are single attribute utility functions. Additive utility, of course, is simply a special case of the multilinear. A second special case, the multiplicative, is presented in Theorem 6.1 of Keeney and Raiffa.

(3) Probabilistic Independence. Just as conditions of independence simplify the modeling of utility they also simplify the modeling of the probability distributions of outcomes. For any particular alternative the attributes may be probabilistically independent. In this case the multivariate probability distribution of outcomes will decompose as a product

$$f(x_1, x_2, \dots, x_n) = f_1(x_1) * f_1(x_2) * \dots * f_n(x_n)$$

of univariate probability distributions, f_i .

(4) Risk Attitude. A single attribute utility function u is often scrutinized by considering lotteries $\langle x, y \rangle$ with outcomes x and y having an equal chance of occurring. The expected value of such a lottery is simply

$$(x + y) / 2$$

The expected utility of this lottery is

$$(u(x) + u(y)) / 2$$

The value z such that $u(z)$ is equal the expected utility is called the certainty equivalent of the lottery.

If a decision maker's utility is such that the certainty equivalent and the expected value are equally preferred, the decision maker is said to be risk neutral. In this case the utility function is linear. If the expected value is preferred to the certainty equivalent, the decision maker is risk averse. If the certainty equivalent is preferred to the expected value, the decision maker is risk prone.

(5) Of all the techniques discussed utility theory is the most theoretically sound; however, it is also the most complex. The greater the number of attributes the greater is the complexity. Increasing availability of application software is making this technique more practical. The recently developed IDEA software, Whitfield et al.(1989), is such an example.

t. Cost-effectiveness ratios. Previously cost-effectiveness ratios were viewed from the stand point of value theory. They can also be viewed from the stand point of utility theory. In this case it is assumed that a decision maker's utility can be represented by a ratio utility function U where

$$U(\text{Cost}, \text{Effectiveness}) = \text{Effectiveness} / \text{Cost}$$

The concept of indifference curves can be defined for utility functions as they were for preference functions. The previous discussion of the ratio preference function, its indifference curves, and preference properties one and four hold in the present context also. It was previously noted that the ratio preference function is equivalent to a preference function with an additive decomposition. The ratio utility function is equivalent to a utility function having a multilinear decomposition. That is

$$U(\text{Cost}, \text{Eff}) = \text{Eff} / \text{Cost}$$

$$= u_1(\text{Cost}) + u_2(\text{Eff}) + w * u_1(\text{Cost}) * u_2(\text{Eff})$$

where u_1 and u_2 are single attribute utility functions of cost and effectiveness, respectively.

The ratio utility function is risk neutral for fixed cost and varying effectiveness. For example, with cost fixed at 3 units, the lottery with effectiveness outcomes 6 and 12, $\langle 6, 12 \rangle$, has expected value 9. The expected utility of this lottery is

$$(6/3 + 12/3) / 2 = 3$$

Consequently the certainty equivalent is 9 ($9/3 = 3$). Since the certainty equivalent equals the expected value the risk neutral case holds.

The ratio utility function is risk prone for fixed effectiveness and varying cost. For example, with effectiveness fixed at 4 units, the lottery with cost outcomes 4 and 12, $\langle 4, 12 \rangle$, has expected value 8. The expected utility of this lottery is

$$(4/4 + 4/12) / 2 = 2/3$$

Consequently the certainty equivalent is 6 ($4/6 = 2/3$). Since the certainty equivalent, 6, is preferable to the expected value, 8 (less cost is better) the risk prone case holds.

The ratio utility function also exhibits decreasing risk proneness with increasing cost, again for fixed effectiveness. This means that the difference between the certainty equivalent and the expected value decreased with increasing cost. Continuing the last example, with effectiveness fixed at 4 units, the lottery with cost outcomes 12 and 20, $\langle 12, 20 \rangle$ has expected value 16. The expected utility of this lottery is

$$(4/12 + 4/20) / 2 = 4/15$$

Consequently the certainty equivalent is 15. The certainty equivalent, 15, is preferable to the expected value, 16, again indicating risk proneness. The difference between the two values is now only 1, whereas in the previous lottery ($\langle 4, 12 \rangle$) over smaller values of cost the difference was 2. This illustrates decreasing risk proneness with increasing cost.

For the ratio utility function the attributes of cost and effectiveness have a complementary effect. This is illustrated with the following two lotteries, L1 and L2.

L1: $\langle (4, 12), (12, 4) \rangle$

L2: $\langle (4, 4), (12, 12) \rangle$

Lottery L1 yields an outcome with low cost and high effectiveness or an outcome with high cost and low effectiveness. Lottery L2, on the other hand, yields an outcome with low cost and low effectiveness or an outcome with high cost and high effectiveness. Lottery L1 with expected utility, $5/3$, is preferred to lottery L2 with expected utility, 1. Preference of L1 indicates that a less preferred level on one attribute cannot be made up with a more preferred level on the other attribute. That is the attributes are complementary as opposed to supplementary (see page 240 of Keeney and Raiffa).

As was the situation with the value theory application the advantage of cost-effectiveness ratios is that the decision maker is required to provide little or no information. The disadvantage is that the decision maker must accept the unique properties that this technique requires.

Appendix C

COEA Data

1. COEA decision criteria. Table C-1 lists a very general set of COEA decision criteria. Refinements of this set yielded the attributes used in the demonstrations.

Table C-1
COEA Decision Criteria

- Combat Effectiveness
 - Threats
 - Scenarios
 - Europe
 - Southwest Asia
 - Type engagement
 - Offense
 - Defense
 - Battlefield Conditions
 - Night or Day
 - Weather
 - Obscurants
 - Attrition
 - Red systems killed
 - Blue systems killed
- Supportability
 - Transportation
 - Air
 - Land
 - Sea
 - Maintenance
 - Availability
 - Reliability
 - Maintainability
 - Recovery
 - Resupply
 - Fuel
 - Ammunition
 - Spare parts
- MANPRINT
 - Manpower
 - Personnel
 - Training
 - System Safety
 - Health Hazards
 - Human Factors Engineering
- Cost (twenty year peacetime)
 - Development Costs
 - Production Costs
 - Military Construction Costs
 - Fielding Costs
 - Sustainment Costs

2. COEA combat scenarios descriptions. Benefits of alternatives were assessed using attrition measures from a high resolution combat simulation model. Three different scenarios were represented. In the three scenarios: European meeting engagement, European defense, and Southwest Asian (SWA) meeting engagement, 1996 blue forces met 2004 red forces. Each scenario is now described.

a. European Brigade Meeting Engagement. In this scenario a balanced blue brigade meets two red reinforced tank battalions in open terrain beginning at mid-morning of a winter day. In this long range mounted battle heavy missile systems and tanks dominate. Artillery effects are minimal. Table C-2 lists the weapons systems and their numbers.

Table C-2
European Brigade Meeting Engagement
Weapons Systems

	Blue	Red
Weapons System		
Tanks	116	56
Infantry Fighting Vehicles	108	38
Dedicated Anti-Tank Systems	24	0
Air Defense Artillery	17	12
Helicopters	16	10
Other	24	10
Total Systems	305	126

b. European Balanced Task Force Defense. In this scenario a balanced mechanized infantry task force defends against two red reinforced tank regiments in open terrain beginning just before dawn on a winter day. Blue forces are presented with abundant fully exposed moving targets at long ranges. Artillery plays a major role on both sides. Table C-3 lists the weapons systems and their numbers.

Table C-3
European Balanced Task Force Defense
Weapons Systems

	Blue	Red
Weapons System		
Tanks	28	188
Infantry Fighting Vehicles	30	116
Dedicated Anti-Tank Systems	12	4
Air Defense Artillery	6	30
Helicopters	16	8
Other	14	34
Total Systems	106	380

c. Southwest Asian (SWA) brigade meeting engagement. In this scenario a blue tank heavy brigade meets a red reinforced tank brigade in open desert terrain during a summer day. Both forces are moving and fully exposed. Engagements are at long ranges and artillery is heavily used. Table C-4 lists the weapons systems and their numbers.

Table C-4
SWA Brigade Meeting Engagement
Weapons Systems

	Blue	Red
Weapons System		
Tanks	116	110
Infantry Fighting Vehicles	54	62
Dedicated Anti- Tank Systems	12	12
Air Defense Artillery	14	15
Helicopters	8	20
Other	52	46
Total Systems	256	265

3. COEA cost data. Table C-5 gives cost estimates of the alternatives. Estimates are in million FY92 constant dollars.

Table C-5
COEA Cost Estimates

Alternative	Cost Estimate
Base Case	350
Alt 1	900
Alt 2	982
Alt 3	1344

4. COEA loss exchange ratio (LER) data. For each alternative and each scenario statistics below summarize the results from 21 replications of a high resolution combat model.

Table C-6
European Brigade Meeting Engagement
LER Estimates

Alternative	Median	Mean	St.Dev	Min	Max
Base Case	1.00	1.06	0.20	0.79	1.31
Alt 1	1.04	1.08	0.18	0.81	1.40
Alt 2	1.40	1.45	0.19	1.10	1.62
Alt 3	1.77	1.81	0.21	1.23	2.01

Table C-7
European Balanced Task Force Defense
LER Estimates

Alternative	Median	Mean	St.Dev	Min	Max
Base Case	2.29	2.35	0.30	1.99	3.16
Alt 1	2.44	2.46	0.24	2.11	3.03
Alt 2	2.38	2.36	0.32	1.87	3.38
Alt 3	2.57	2.78	0.48	2.15	3.88

Table C-8
SWA Brigade Meeting Engagement
LER Estimates

Alternative	Median	Mean	St.Dev	Min	Max
Base Case	1.34	1.30	0.24	0.99	1.73
Alt 1	1.37	1.40	0.19	0.98	1.74
Alt 2	1.49	1.52	0.18	1.16	1.88
Alt 3	1.62	1.63	0.17	1.26	2.05

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